Non-Asymptotic Analysis of Single-Receiver Channels with Limited Feedback

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Outline

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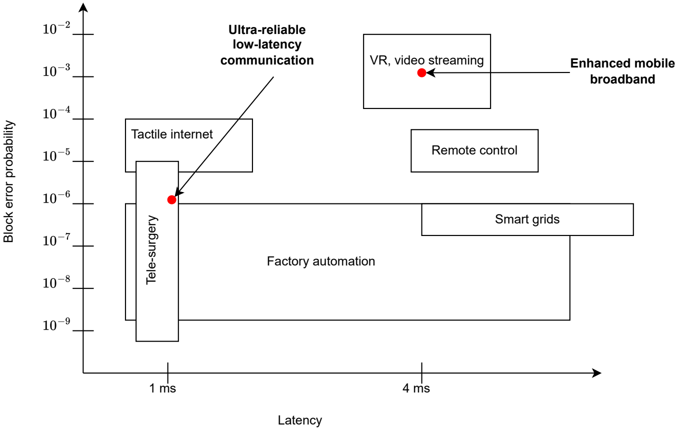
1. Motivation
2. Dissertation
   - Moderate Deviations Analysis of Point-to-Point Channels
   - Variable-Length Sparse Stop-Feedback Codes
   - Random Access Channels with Sparse Stop-Feedback
3. Conclusion and Future Directions

5G and newer generation networks offer three new service categories

- Ultra-reliable low-latency communication
  - Delay-sensitive, mission critical applications (tele-surgery, factory automation)
- Enhanced mobile broadband
  - High bandwidth and low power consumption requirements (HD video streaming, virtual reality)
- Massive machine-type communication
  - Massive and unknown number of communicating devices with low power consumption (Internet of Things, smart cities, smart grids)

Goal: Design coding schemes that are suitable for applications like these, analyze their performance, and understand the fundamental limits from an information-theoretic perspective.

Applications with Stringent Requirements



1 ms latency $\approx n = 100$–200

M. Shirvanimoghaddam et al., "Short Block-Length Codes for Ultra-Reliable Low Latency Communications", IEEE Commun. Mag., Feb. 2019.

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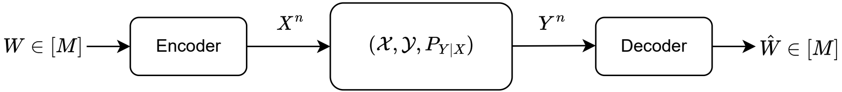
Channel Coding Problem



- M = codebook size, n = blocklength, ϵ = average error probability, $[M] \triangleq \{1, \ldots, M\}$.
- Fundamental limit: $M^*(n, \epsilon)$ = maximum achievable codebook size compatible with n and ϵ
- Goal: Compute $M^*(n, \epsilon)$ and find the optimal codes that achieve it
- Problem: Exact computation of $M^*(n, \epsilon)$ is intractable
- Solution: Derive non-asymptotic bounds

$$\underline{M}^*(n, \epsilon) \leq M^*(n, \epsilon) \leq \overline{M}^*(n, \epsilon)$$

- Example of non-asymptotic bound [Polyanskiy 10', random coding union bound]:

$$\underline{M}^*(n,\epsilon) = \max\{M : \mathbb{E}\left[\min\{1, (M-1)\mathbb{P}\left[\imath(\bar{X}^n; Y^n) \geq \imath(X^n; Y^n) | X^n, Y^n\right]\}\right] \leq \epsilon\}$$

  The "solution" is an optimization problem
  - it does not make clear how $M^*(n,\epsilon)$ varies with $(P_{Y|X}, n, \epsilon)$
  - even approximately computing it is nontrivial

- We are interested in asymptotic expansions (Taylor-like expansions of $\frac{\log M^*(n,\epsilon)}{n}$ around capacity) that
  - are tight for $(P_{Y|X}, n, \epsilon)$ of interest
  - inform about how $M^*(n,\epsilon)$ varies with $(P_{Y|X}, n, \epsilon)$
  - cover a variety of channels

- Analyzing $M^*(n,\epsilon)$ as $n \to \infty$ is useful!

Error Probability Regimes in Channel Coding

As $n \to \infty$, we need to specify how ϵ is related to n

- **Central limit theorem (CLT) regime:**

$$\begin{aligned} &\epsilon \in (0,1) \\ &\log M^*(n,\epsilon) = nC - O(\sqrt{n}) \\ &C = \text{capacity} \end{aligned}$$

- **Moderate deviations (MD) regime**

$$\epsilon \to 0 \text{ and } -\frac{1}{n}\log\epsilon \to 0 \qquad \log M^*(n,\epsilon) = nC - g(n), \quad \sqrt{n} \ll g(n) \ll n$$

- **Large deviations (LD) regime:**

$$\begin{aligned} &\epsilon \approx e^{-nE(R)} \\ &\log M^*(n,\epsilon) = nR, \quad R < C \end{aligned}$$

MD regime is relevant to ultra-reliable short-blocklength regime where both ϵ and n are small.

CLT and LD regimes fail to capture this regime.

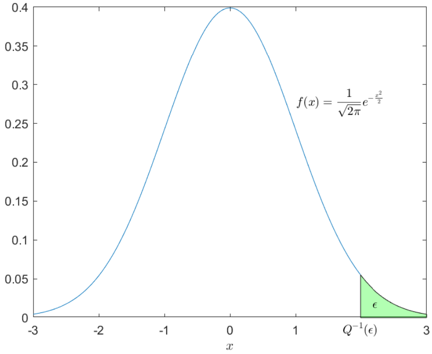
- [Strassen 62'] and [Polyanskiy et al. 10'] show

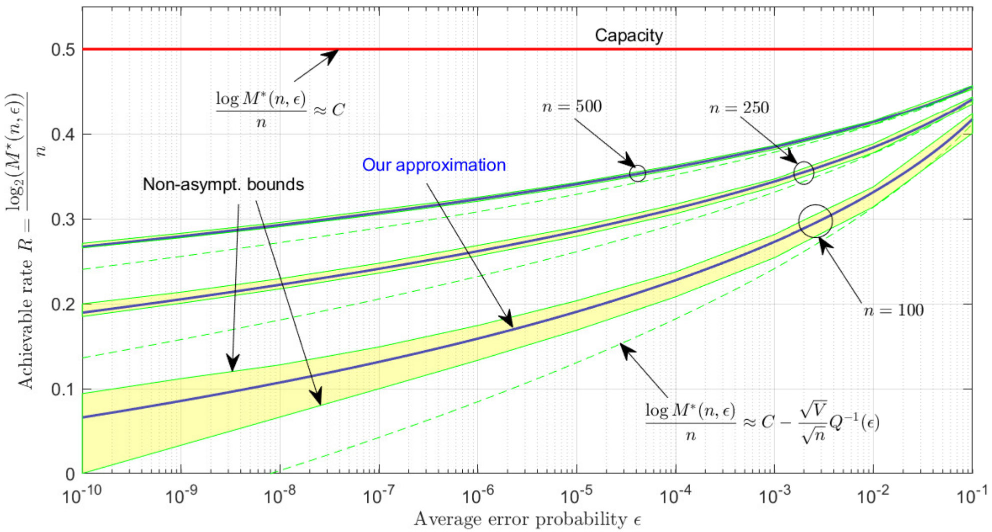
$$\log M^*(n,\epsilon) = \underbrace{nC - \sqrt{nV}Q^{-1}(\epsilon)}_{\text{Gaussian approximation}} + O(\log n)$$

C = capacity

V = dispersion (a quantity that depends only on $P_{Y|X}$)

$Q^{-1}(\cdot)$ = inverse of $Q(\cdot)$, complementary cumulative distribution function of $\mathcal{N}(0,1)$





Achievable rates for BSC(0.11)

Non-Gaussianity:

$$\zeta(n,\epsilon) \triangleq \log M^*(n,\epsilon) - \underbrace{\left(nC - \sqrt{nV}Q^{-1}(\epsilon)\right)}_{\text{Gaussian approximation}}$$

1. **CLT**: [Polyanskiy et al. 10'] (achievability) and [Tomamichel-Tan 13'] (converse): for $\underbrace{\text{nonsingular channels}}_{\text{excludes a family of erasure channels}}$

$$\zeta(n,\epsilon) = \frac{1}{2}\log n + O(1)$$

2. **CLT**: [Moulin 17']: for nonsingular channels with a regularity condition (excludes the BSC)

$$\zeta(n,\epsilon) \geq \frac{1}{2}\log n + \underline{S}Q^{-1}(\epsilon)^2 + \underline{B} + o(1)$$
$$\zeta(n,\epsilon) \leq \frac{1}{2}\log n + \overline{S}Q^{-1}(\epsilon)^2 + \overline{B} + o(1)$$

where $\underline{S}, \overline{S}, \underline{B}$, and $\overline{B}$ depend only on $P_{Y|X}$

- **MD**: [Altuğ and Wagner 14']:

$$\zeta(n, \epsilon_n) = o(\sqrt{n} Q^{-1}(\epsilon_n))$$

Gaussian approximation (without $O(\log n)$ term) is valid in the MD regime

- **Question:** How does the non-Gaussianity $\zeta(n, \epsilon_n)$ scale in the MD regime?

Channel Skewness

- A new operational quantity: Channel skewness

$$S \triangleq \lim_{\epsilon \to 0} \liminf_{n \to \infty} \frac{\zeta(n,\epsilon) - \frac{1}{2}\log n}{(Q^{-1}(\epsilon))^2}$$

We are expecting:

$$\log M^*(n,\epsilon_n) = nC - \sqrt{nV}Q^{-1}(\epsilon_n) + \frac{1}{2}\log n + SQ^{-1}(\epsilon_n)^2 + o(Q^{-1}(\epsilon_n)^2)$$

- Information density: $\imath(X;Y) \triangleq \log \frac{P_{Y|X}(Y|X)}{P_Y(Y)}$ under some fixed input distribution P_X

$$\left.\begin{array}{ll} C & = \mathbb{E}\left[\imath(X;Y)\right] \\ V & = \mathrm{Var}\left[\imath(X;Y)\right] \end{array}\right\} \text{ under capacity-achieving } P_X$$

$$S = \text{governed by the third central moment of information density}$$

Our Bounds on the non-Gaussianity in the MD Regime

ϵ_n in MD regime: $-\frac{1}{n}\log\epsilon_n \to 0$, $\epsilon_n \to 0$

Theorem 1

MD: *for nonsingular discrete memoryless point-to-point channels*

$$\zeta(n,\epsilon_n) \geq \frac{1}{2}\log n + \underline{S}\, Q^{-1}(\epsilon_n)^2 + O\left(\frac{Q^{-1}(\epsilon_n)^3}{\sqrt{n}}\right) + O(1)$$

$$\zeta(n,\epsilon_n) \leq \frac{1}{2}\log n + \overline{S}\, Q^{-1}(\epsilon_n)^2 + O\left(\frac{Q^{-1}(\epsilon_n)^3}{\sqrt{n}}\right) + O(1)$$

- $\underline{S}$ and $\overline{S}$ are the same as Moulin's bounds, and our results apply to a wider class of channels
- $\underline{S}$ and $\overline{S}$ are easy-to-compute constants that depend only on the channel $P_{Y|X}$ and are related to $\imath(X;Y)$
- $\underline{S}$ and $\overline{S}$ give lower and upper bounds for channel skewness S
- $\frac{1}{2}\log n + O(1)$ is no longer accurate because as $n\to\infty$, $\epsilon_n \to 0$ and $Q^{-1}(\epsilon_n)^2 \to \infty$

- Lower bound is achieved using i.i.d. random codewords drawn from a distribution P_X that is shifted from the capacity-achieving P_X^*

$$P_X = P_X^* + \frac{Q^{-1}(\epsilon_n)}{\sqrt{n}}\mathbf{h}$$

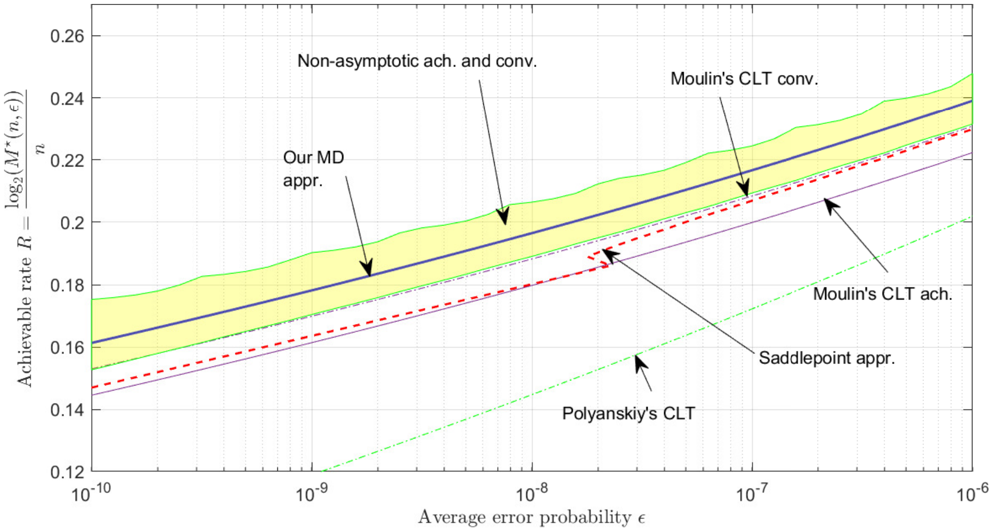
$\mathbf{h}$ = a vector that is a function of $P_{Y|X}$

- For Cover-Thomas symmetric channels (all rows (resp. columns) are permutation of each other), e.g., BSC

$$S = \overline{S} = \underline{S} = \frac{\mathrm{Sk}(P_X^*)\sqrt{V}}{6} + \frac{1}{2}$$
$$P_X = P_X^*$$
$$\mathrm{Sk}(P_X^*) = \text{skewness of } \imath(X;Y) \text{ at } P_X^*$$

- For the Gaussian channel, we compute the channel skewness $S(P)$ exactly as a function of power P

Comparison of Our Approximation to Existing Work



Achievable rates for BSC(0.11), $n = 200$.

- We relax the random coding union bound from [Polyanskiy et al. 10'] as

$$P_{\text{error}} \leq \mathbb{E}\left[\min\{1, (M-1)\mathbb{P}\left[\imath(\bar{X}^n; Y^n) \geq \imath(X^n; Y^n) | X^n, Y^n\right]\}\right]$$
$$\leq \mathbb{P}\left[\imath(X^n; Y^n) < \tau\right] + (M-1)\mathbb{P}\left[\imath(\bar{X}^n; Y^n) \geq \imath(X^n; Y^n) \geq \tau\right]$$

where $\bar{X}^n$ is a sample from the codebook that is independent of Y^n.

  - Relaxation of the ML decoder (information density threshold rule + ML rule)
  - Advantageous because we can analyze each probability term separately

Achievability Proof

- The optimal allocation of ϵ_n (equivalently, the optimal τ) is:

$$\underbrace{\mathbb{P}\left[\imath(X^n;Y^n) < \tau\right]}_{\epsilon_n - \epsilon_n \frac{Q^{-1}(\epsilon_n)}{\sqrt{nV(P_X)}}} + \underbrace{(M-1)\mathbb{P}\left[\imath(\bar{X}^n;Y^n) \geq \imath(X^n;Y^n) \geq \tau\right]}_{\epsilon_n \frac{Q^{-1}(\epsilon_n)}{\sqrt{nV(P_X)}}}$$

- $\mathbb{P}\left[\imath(X^n;Y^n) < \tau\right]$: MD regime of probability theory
  $\mathbb{P}\left[\imath(\bar{X}^n;Y^n) \geq \imath(X^n;Y^n) \geq \tau\right]$: LD regime of probability theory

Moderate Deviations Theorem

Theorem 2 (Petrov Expansion)

Let $X_1, \ldots, X_n$ *be i.i.d.,* $\mathbb{E}[X_1] = 0$ *and* $\mathrm{Var}[X] = \sigma^2$. *Let* $x = o(\sqrt{n})$, *and* $x \geq 0$. *Under Cramér's condition, i.e.,* $\mathbb{E}\left[e^{tX}\right] < \infty$ *in the neighborhood of zero,*

$$\mathbb{P}\left[\sum_{i=1}^{n} X_i \geq \sqrt{n\sigma^2}x\right] = Q(x)\exp\left(\frac{x^3 \mathrm{Sk}(X_1)}{6\sqrt{n}} + O\left(\frac{x^4}{n}\right)\right)\left(1 + O\left(\frac{1+x}{\sqrt{n}}\right)\right)$$

where $\mathrm{Sk}(X_1) \triangleq \frac{\mathbb{E}[X_1^3]}{\sigma^3}$.

- It is an asymptotic equality
- It has a multiplicative correction term to Gaussian approximation $Q(x)$
- Skewness $\mathrm{Sk}(X_1)$ dominates the correction term

- $\mathbb{P}\left[\imath(X^n;Y^n) < \tau\right]$: Apply Petrov expansion
  $\mathbb{P}\left[\imath(\bar{X}^n;Y^n) \geq \imath(X^n;Y^n) \geq \tau\right]$: Apply strong large deviations theorem to get

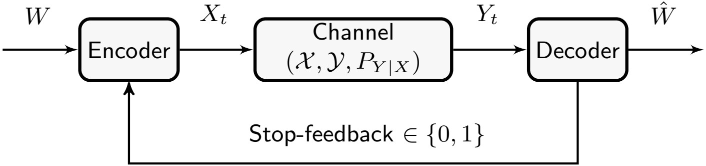
$$\log M^* \geq nI(P_X) - \sqrt{nV(P_X)}Q^{-1}(\epsilon_n) + \frac{1}{2}\log n$$
$$+ Q^{-1}(\epsilon_n)^2\left(\frac{\mathrm{Sk}(P_X)\sqrt{V(P_X)}}{6} + \frac{1-\eta(P_X)}{2(1+\eta(P_X))}\right) + O\left(\frac{Q^{-1}(\epsilon_n)^3}{\sqrt{n}}\right) + O(1)$$

- Take Taylor series expansions around the capacity-achieving P_X^*.

Summary of Part 1

1. In the MD regime of channel coding:
   - We derive easy-to-compute lower and upper bounds to the non-Gaussianity
   - For symmetric channels, the bounds match each other, and channel skewness S is governed by the skewness of information density $\imath(X;Y)$
   - Our MD approximation is more accurate than the CLT approximation of Polyanskiy, especially for lower error probabilities (e.g., $\epsilon \leq 10^{-6}$)
2. For some non-symmetric channels, capacity-achieving distribution does not achieve S.
3. We compute the skewness of the Gaussian channel.

Outline



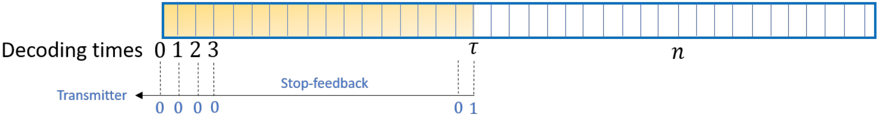
Variable-length: Decoding occurs at a random time depending on channel outputs

Stop-feedback: 1 bit feedback whenever a decoding attempt is made

Higher reliability: Earlier decoding when the noise is low, later decoding when the noise is high

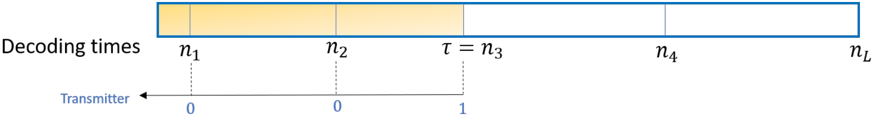
Why Sparse Stop-Feedback?

- VLSF code with $L = \infty$ decoding times [Polyanskiy et al. 11'], [Burnashev 76']: Impractical

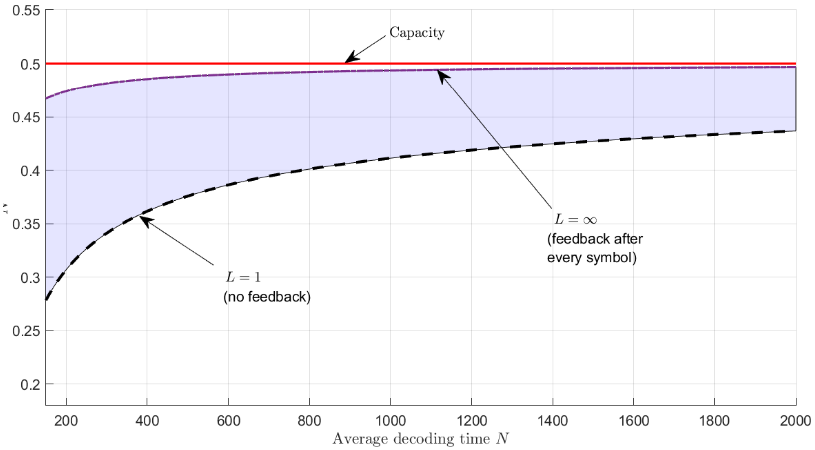


  - Transmitter constantly listens to the feedback signal $\Longrightarrow$ High power consumption
  - Half duplex devices cannot transmit and receive signals at the same time $\Longrightarrow$ Lowers achievable rates due to round trip delay
  - Practical codes such has HARQ schemes employ incremental redundancy which has sporadic feedback

- VLSF code with constant L decoding times (this work)

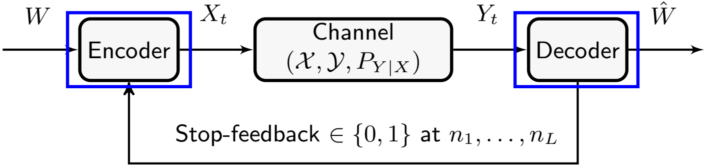


  - Feedback is sporadic



$L = \#$ of available decoding times

What rate can we achieve with several decoding times?

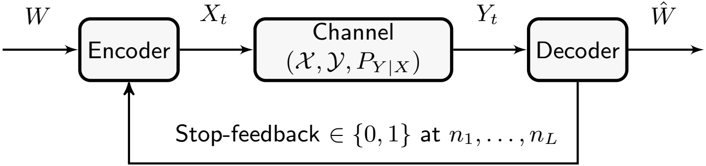


Encoding function $\mathsf{f}_t \colon [M] \to \mathcal{X}$:

$$X_t = \mathsf{f}_t(W), \quad t \in \mathbb{N}_+$$

where $W \sim \mathrm{Unif}\,([M])$.

Decoding function $\mathsf{g}_t \colon \mathcal{Y}^t \to [M]$: provides the estimate of W at time $t \in \{n_1, n_2, \ldots, n_L\}$.



Stopping time: $\tau \in \{n_i\}_{i=1}^{L}$ adapted to the filtration generated by $\{Y^{n_i}\}_{i=1}^{L}$

Decoded message: $\hat{W} = \mathrm{g}_\tau(Y^\tau)$

Goal: Find

$$\begin{aligned} M^*(N, L, \epsilon) \triangleq \max_{n_1, \ldots, n_L} \quad & M \\ \text{s.t.} \quad & \mathbb{E}[\tau] \leq N \\ & \mathbb{P}\left[\hat{W} \neq W\right] \leq \epsilon \end{aligned}$$

$L = \infty$:

- [Burnashev 76']: error exponent $\lim\limits_{\epsilon \to 0} \frac{-\log \epsilon}{\mathbb{E}[\tau]}$ for the discrete memoryless point-to-point channel as $N \to \infty$
- [Polyanskiy et al. 11']: VLSF codes in the CLT regime

$$\frac{NC}{1-\epsilon} - \log N + O(1) \leq \log M^*(N, \infty, \epsilon) \leq \frac{NC}{1-\epsilon} + O(1)$$

$$\log M^*(N, 1, \epsilon) = NC - \sqrt{NV} Q^{-1}(\epsilon) + O(\log N)$$

Constant L**:**

- [Vakilinia et al. 16']: VLSF codes with L decoding times over the binary-input Gaussian channel
- They estimate the statistics of τ through simulation.
- They do not solve the problem analytically $\Longrightarrow$ no second-order analysis

Theorem 3 (Achievability)

Fix $L \geq 2$, $\epsilon \in (0,1)$. For discrete memoryless point-to-point channels, it holds that

$$\log M^*(N, L, \epsilon) \geq \frac{NC}{1-\epsilon} - \sqrt{N \log_{(L-1)}(N) \frac{V}{1-\epsilon}} + o(\sqrt{N})$$

$$\log_{(L)}(\cdot) \triangleq \overbrace{\log(\log(\ldots(\log(\cdot))))}^{L \text{ times}}$$

- Proof analyzes a non-asymptotic bound
- By using refined probability tools and KKT conditions, we approximately optimize the non-asymptotic bound with respect to the choices of $n_1, \ldots, n_L$

Performance of VLSF Codes with L Decoding Times

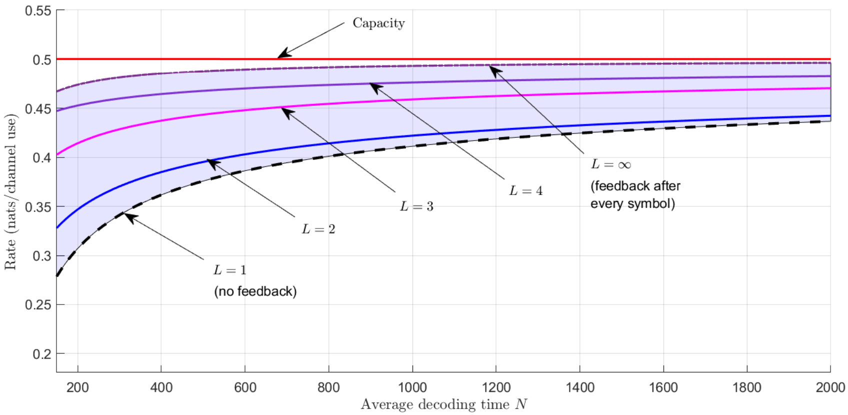


Figure 1: BSC(0.11), $\epsilon = 10^{-3}$

Diminishing performance improvement as L increases!

Sequential Hypothesis Tests (SHTs)

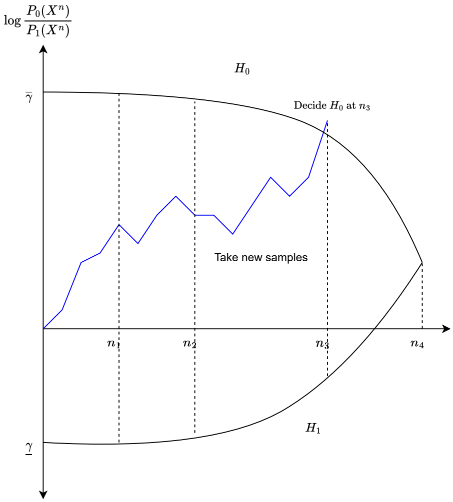
- Consider two hypotheses for the distribution of X^∞

$$H_0 : X^\infty \sim P_0^\infty$$
$$H_1 : X^\infty \sim P_1^\infty$$

- At each available decoding time, an SHT chooses between

$$\{\text{Decide } H_0, \text{Decide } H_1, \text{Take new sample(s)}\}$$

- Wald's theorem: the optimal SHT is a two-sided threshold test that uses log-likelihood ratio (sequential version of Neyman-Pearson lemma)



- Construct the SHT

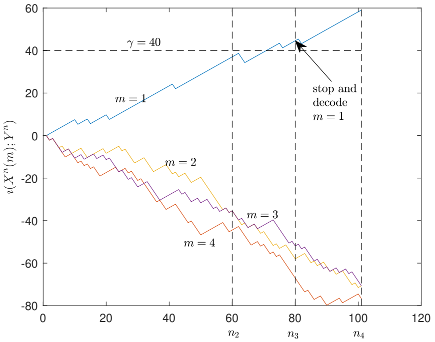
$H_0 : (X,Y) \sim P_X P_{Y|X} \Longrightarrow$ stop and decode

$H_1 : (X,Y) \sim P_X P_Y \Longrightarrow$ eliminate from decoding

where P_X is capacity-achieving and run it for all M messages separately

- At time $n_1 = 0$:
  - With probability $p = \epsilon - \frac{1}{\sqrt{N \log N}}$, declare H_1 for all $m \in [M]$
  - With probability $1 - p$, pass $n_1 = 0$ without decoding
- If $n_1 = 0$ is passed, then particularize the SHT to information-density threshold test:

$$\tau_m = \min\{n \in \{n_2, \dots, n_L\} : \imath(X^n(m); Y^n) \geq \gamma\}$$



- Analyzing this scheme gives us a non-asymptotic bound. Then, we optimize over $n_1, \dots, n_L$ using KKT conditions.

Maximal power constraint:

$$\|\mathsf{f}(m)^{n_\ell}\|_2^2 \leq n_\ell P \qquad \forall \ell \in [L], m \in [M]$$

Theorem 4

Fix $L \geq 2$, $P > 0$, *and* $\epsilon \in (0,1)$.

$$\log M^*_{\max}(N, L, \epsilon, P) \geq \frac{NC(P)}{1-\epsilon} - \sqrt{N \log_{(L-1)}(N) \frac{V(P)}{1-\epsilon}} + o(\sqrt{N})$$

where

$$C(P) = \frac{1}{2}\log(1+P), \quad V(P) = \frac{P(P+2)}{2(P+1)^2}$$

- **Encoder:** we generate codewords uniformly at random over a restricted subset on n_L-dimensional sphere

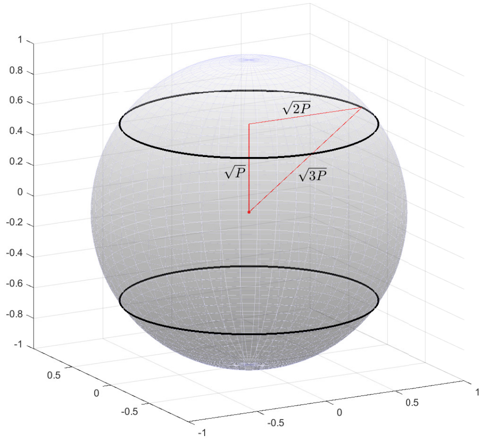
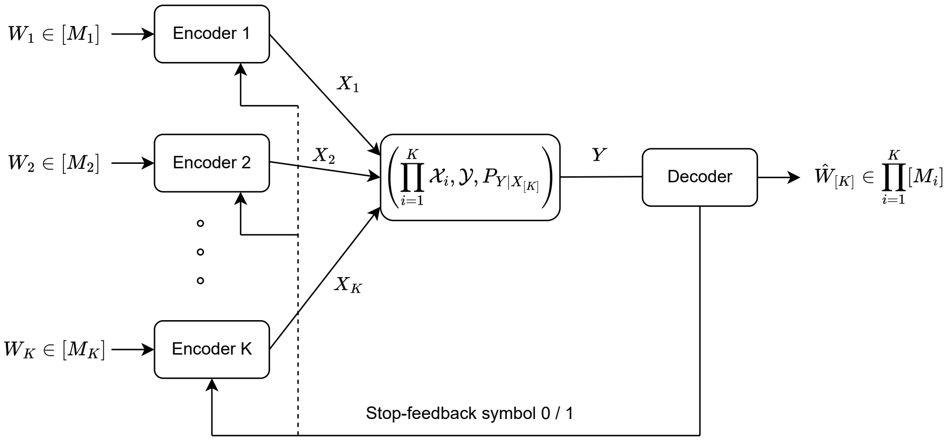


Figure 2: $L = 2$, $n_1 = 2, n_2 - n_1 = 1$



- $\{n_1, \ldots, n_L\}$ = the set of available decoding times
- $\tau \in \{n_1, \ldots, n_L\}$ = stopping time
- $X_{[K]} \triangleq X_1, \ldots, X_K$

Goal: Find

$$\begin{aligned} \max_{n_1,\dots,n_L} \quad & \sum_{k=1}^{K} \log M_k \\ \text{s.t.} \quad & \mathbb{E}\left[\tau\right] \leq N \\ & \mathbb{P}\left[(\hat{W}_1,\dots,\hat{W}_K) \neq (W_1,\dots,W_K)\right] \leq \epsilon \end{aligned}$$

Theorem 5 (Achievability)

There exists a VLSF code with L decoding times for the discrete memoryless MAC satisfying

$$\sum_{k=1}^{K} \log M_k = \frac{N\, I_K}{1-\epsilon} - \sqrt{N \log_{(L-1)}(N) \frac{V_K}{1-\epsilon}} + o(\sqrt{N})$$

$I_K = I(X_1,\dots,X_K;Y)$, $V_K = \mathrm{Var}\left[\imath(X_1,\dots,X_K;Y)\right]$

- Proof: We employ a single information density threshold decoder $\imath(X_{[K]}^{n_\ell}(m_{[K]}); Y^{n_\ell}) \geq \gamma$.

Summary of Part 2

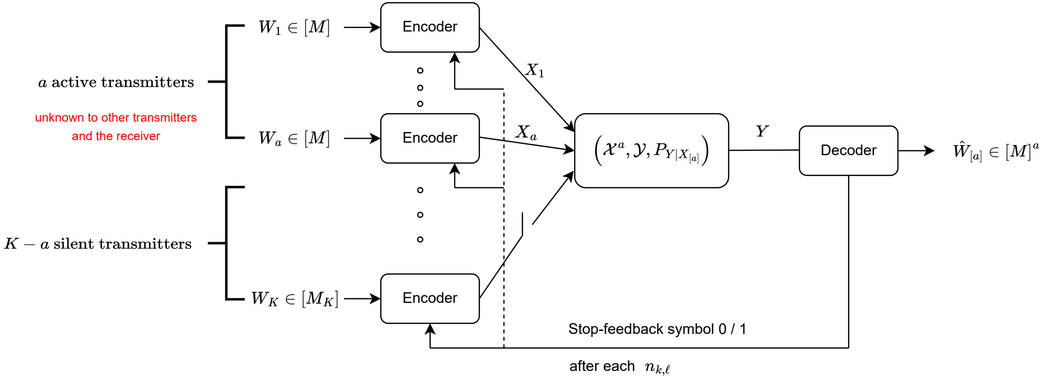
- Second-order achievability bounds for sparse VLSF codes over point-to-point and multiple access channels

- Optimizing the placements of L decoding times is critical to achieve high rates

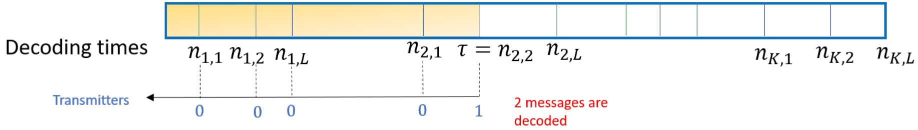
- A handful of decoding times is almost as good as decoding after every symbol
  For the BSC(0.11) at $N = 1000$ and $\epsilon = 10^{-3}$:
  - $L = 4$ achieves 97.0% of the rate achieved by $L = \infty$
  - $L = 1$ achieves 84.2% of the rate achieved by $L = \infty$

Outline

Rateless RAC (Random Access Channel) Communication that Uses Stop-Feedback



- **RAC**: A family of MACs up to K transmitters, any $a \leq K$ transmitters can be active
- **Compound channel model:** No probability of being active is assigned to transmitters
- **Agnostic channel model:** Nobody knows who are active
- We extend VLSF codes to the RAC
- Available decoding times $= n_{a,1}, \ldots, n_{a,L}$ for decoding $a \in [K]$ messages



Goal: Find

$$\max_{n_{a,\ell}\,:\, a\in[K],\ell\in[L]} M$$
$$\text{s.t.} \quad \mathbb{E}\left[\tau_a\right] \leq N_a \qquad a \in [K]$$
$$\mathbb{P}\left[(\hat{W}_1, \ldots, \hat{W}_a) \neq (W_1, \ldots, W_a)\right] \leq \epsilon_a \qquad a \in [K]$$

- Presented in the candidacy talk.

Theorem 6 (Achievability)

Fix an input distribution P_X. For a RAC satisfying some mild symmetry conditions, there exists a RAC code with K transmitters provided that

$$a \log M \leq N_a I_a - \sqrt{N_a V_a} Q^{-1}(\epsilon_a) - \frac{1}{2} \log N_a + O(1) \qquad \forall\, a \in [K]$$

We achieve the same first- and second-order terms as the best-known codes (e.g., [Tan-Kosut 14'], [Scarlett et al. 15]) for the MAC in operation.

- Proof: At times $n_{1,1}, \ldots, n_{K,1}$, we use information density threshold rule

- Presented in the candidacy talk.

Theorem 7

There exists a RAC code for the Gaussian RAC with K transmitters provided that

$$a \log M \leq N_a C(aP) - \sqrt{N_a V_a(P)} Q^{-1}(\epsilon_a) + \frac{1}{2} \log N_a + O(1) \qquad \forall\ a \in [K]$$

where

$$C(P) = \frac{1}{2} \log(1+P), \quad V_a(P) = \frac{(2a^2 - a)P^2 + 2aP}{2(aP+1)^2}$$

- Proof analyzes the error probability of the random code where codewords are generated uniformly on the restricted power sphere

Theorem 8 (Achievability)

Fix K, $L \geq 2$, $\epsilon \in (0,1)$, and a distribution P_X. Under some mild symmetry conditions, there exists a VLSF code for the discrete memoryless RAC with L decoding times for each $a \in [K]$ provided that

$$a \log M \leq \frac{N_a I_a}{1-\epsilon_a} - \sqrt{N_a \log_{(L-1)}(N_a) \frac{V_a}{1-\epsilon_a}} + o(\sqrt{N_a}) \qquad \forall\, a \in [K]$$

We achieve the same first- and second-order terms as the MAC in operation

- **Decoder:** At time n_0, the decoder applies a multiple hypothesis test to estimate # of active transmitters a

  If $\hat{a}$ is decoder's estimate, decoder uses a VLSF code for the $\hat{a}$-MAC with decoding times $\{n_{\hat{a},1}, \ldots, n_{\hat{a},L}\}$

Outline

Take-home message: Part 1

- In the low-latency, high-reliability regime, our MD approximation is more accurate than several state-of-the-art approximations
- In this regime, channel skewness cannot be neglected to get tight approximations

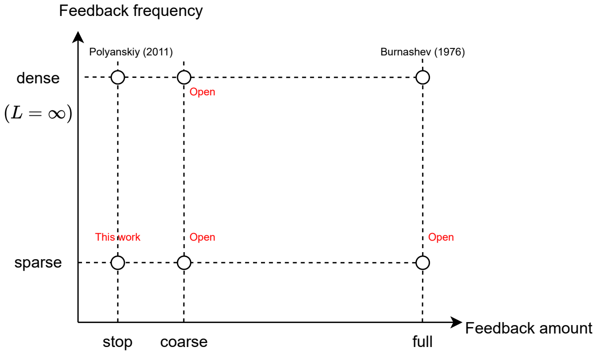
Take-home message: Part 2

- A handful of decoding times in VLSF codes achieve rates close to those achieved by $L = \infty$

Take-home message: Part 3

- For RACs, we achieve the same first- and second-order rates as if the active transmitters are known a priori
- Rateless coding with stop-feedback enables this result

- **Different ways of limiting the feedback in VLSF codes:**



  - Coarse feedback: Send R_f bits of feedback at each time
  - In many applications, we can send bursty feedback

- **A tight converse for VLSF codes with L decoding times** is missing

- Towards this goal, we have a non-asymptotic converse bound based on fundamental limits of SHTs
  This bound seems to be very challenging to analyze

- **Second-order converse for the RAC result:**
  - A second-order converse result for the K-MAC is also a converse result for the RAC
  - However, it is a long-standing open problem
  - [Kosut 22'] proves that the second-order term scales as $-O(\sqrt{N})$

$$K \log M^* = n_K I_K - O(\sqrt{n_K})$$

Thank you!

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